SAMPLE-BALANCE ANALYSIS OF NONLINEAR AUTONOMOUS CIRCUITS

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Abstract — This paper presents a robust approach for the analysis of nonlinear microwave autonomous circuits. A new formulation of the Sample-Balance (SB) equation for autonomous circuits completely suppresses the undesirable degenerate (DC) solution. Three techniques are combined to solve the SB equation: "signal synchronization", "harmonic stepping", and pseudo arc-length continuation. An example illustrates the excellent convergence properties obtained.

I. INTRODUCTION

The analysis of nonlinear microwave autonomous circuits by steady-state frequency-domain methods has received considerable attention in the 1990s [1–7]. When compared to the analysis of forced circuits, at least two additional problems exist. First, the frequency of oscillation is not known in advance. To accommodate this extra variable, the standard procedure is to set the phase of a Fourier coefficient of one of the circuit variables to an arbitrary value. A consequence of this procedure is that the DC solution (often called *degenerate* solution) satisfies the circuit equations. Hence, it is necessary to prevent convergence to this undesirable solution. The other problem is finding the wanted solution. Methods for solving systems of nonlinear algebraic equations are only assured to converge when a good estimate of the solution is available. In forced circuits, source stepping can always be used to go from the DC solution to a highly nonlinear situation by solving a sequence of local problems. A similar strategy is not possible for autonomous circuits. Instead, the solution corresponding to a highly nonlinear situation has to be reached directly without a good previous estimate.

A robust approach based on Sample Balance (SB) [8] for the analysis of nonlinear microwave circuits was developed to address the problems discussed in the preceding paragraph. The degenerate solution is completely suppressed by a new formulation of the SB equation. This formulation can be readily adapted to Harmonic Balance (HB) [9]. But, contrary to existing formulations [4,5,7], no singularity is introduced in the frequency-domain circuit equations.

To obtain an initial estimate of the solution, all variables in the circuit are "synchronized" by solving an auxiliary forced circuit using standard techniques. This step is termed *signal synchronization*. Another useful technique is *harmonic stepping*. Whenever convergence problems occur, the SB equations are initially solved for a small number of harmonics where a solution is easier to compute, even though the solution thus obtained is corrupted by aliasing errors. Then, the number of harmonics considered is increased until the desired accuracy is attained. The SB equation is solved by *pseudo arc-length continuation* [9–11] with artificial parameter. In this technique, the desired solution is reached by solving a sequence of local problems. Although the intermediate problems have no physical meaning, convergence properties are greatly improved. To illustrate the use of the approach described in this paper, the simulation of a MESFET oscillator is presented.

II. SUPPRESSION OF THE DEGENERATE SOLUTION

In SB, variables and equations are the inverse discrete Fourier transforms of HB variables and equations [9]. Therefore, SB variables and equations are linearly related to their HB counterparts. Although SB has interesting characteristics regarding scaling of both variables and equations, sparse matrix techniques can only be efficiently employed in HB due to a sparser Jacobian. However, when restricted to full matrix techniques, SB and HB are roughly equivalent.

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Let ω_0 be the unknown fundamental frequency of oscillation of an autonomous circuit. Following [8], the SB equation can be cast in the form

$$\boldsymbol{F}(\boldsymbol{x},\omega_0) = y(\omega_0)\boldsymbol{v}(\boldsymbol{x},\dot{\boldsymbol{x}}) + \boldsymbol{i}(\boldsymbol{x},\dot{\boldsymbol{x}}) - \boldsymbol{u} = \boldsymbol{0} \qquad (1)$$

where $\boldsymbol{x}$ is the vector of time samples of the nonlinear subcircuit controlling variables, $\dot{\boldsymbol{x}}$ contains samples of the first-order derivatives of the controlling variables,[1] $y(\omega_0)$ is a matrix related to the linear subcircuit, $\boldsymbol{v}(\cdot,\cdot)$ and $\boldsymbol{i}(\cdot,\cdot)$ are nonlinear functions that describe port voltages and currents of the nonlinear subcircuit, and $\boldsymbol{u}$ is the vector of samples of Norton equivalent independent (DC) current sources. $\dot{\boldsymbol{x}}$ is a function of both $\boldsymbol{x}$ and ω_0, and can be computed either taking derivatives in the frequency domain or using second (or higher) order backward difference formulas [8,9].

In autonomous circuits, the time reference of steady-state solutions is irrelevant and phase-shifted solutions can be considered equivalent. To accommodate the extra variable ω_0, the standard procedure described in the Introduction is invariably used. To prevent convergence to the degenerate solution, Fourier coefficients of harmonics of waveforms in the circuit are introduced in the denominator of the HB equation [4,5,7]. Although the HB error is likely to grow around the degenerate solution, in such formulation a 0/0 singularity is created at the degenerate solution.

The singularity formulation can give good results, but 0/0 singularities may become troublesome in numerical analysis problems. In the new formulation described here, a sample of a controlling variable, say $x_1(t_0)$, is set to be off the DC value of $x_1(t)$ by δ (Fig. 1), that is,

$$x_1(t_0) = X_1(0) + \delta = \frac{\sum_{k=0}^{N-1} x_1(t_k)}{N} + \delta \qquad (2)$$

where $X_1(0)$ is the DC value of $x_1(t)$ and N is the number of samples. When solved together with (1), this additional equation completely suppresses the degenerate solution if $\delta \neq 0$, without introducing a singularity. From (2),

$$x_1(t_0) = \frac{N\delta + \sum_{k=1}^{N-1} x_1(t_k)}{N-1} \qquad (3)$$

Hence, (3) is actually used to eliminate $x_1(t_0)$ from (1). The choice of δ is not critical. Provided δ is compatible with the power level expected in the circuit, convergence problems were not detected even for small values of δ. The value of δ simply sets a time reference. Typically, δ is set between 0.01 and 0.1 V for voltage variables.

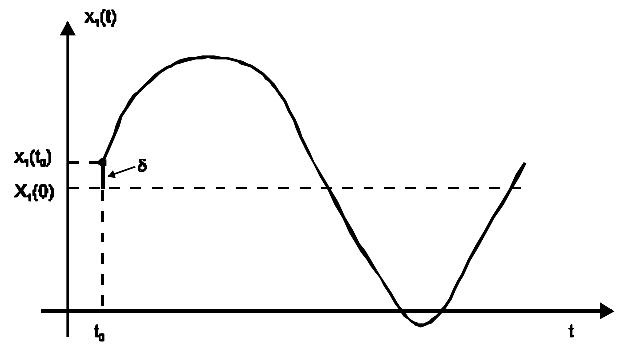

Fig. 1. Illustration of the meaning of (2).

III. Solving the Circuit Equations

Let $\boldsymbol{y}$ be a vector equal to $\boldsymbol{x}$ except that ω_0 replaces $x_1(t_0)$. When (3) is used to eliminate $x_1(t_0)$ from (1), the SB equation can be written as $\boldsymbol{G}(\boldsymbol{y}) = \boldsymbol{0}$. To obtain good convergence properties even when the initial estimate is far from the solution, this equation is solved by a pseudo arc-length continuation method similar to that described in [10]. Because there is no natural continuation parameter in autonomous circuits, a continuation process with artificial parameter α is adopted,

$$\boldsymbol{H}(\boldsymbol{y},\alpha) \triangleq \boldsymbol{G}(\boldsymbol{y}) - (1-\alpha)\boldsymbol{G}(\boldsymbol{y}^0) = \boldsymbol{0} \qquad (4)$$

where $\boldsymbol{y}^0$ is the initial estimate of the solution. Clearly, the solution of (4) for $\alpha = 0$ is $\boldsymbol{y}^0$, while the wanted solution corresponds to $\alpha = 1$. Starting at $(\boldsymbol{y}^0,0)$, the curve in the $(\boldsymbol{y},\alpha)$ space that attends (4) is followed until $\alpha = 1$. The curve is described using the arc-length as parameter so that progress is possible even when α decreases along the curve [9,11]. This characteristic results in excellent

[1] Higher-order derivatives may be considered if required.

convergence properties. Each point in the curve is computed by solving a local problem using Newton´s method. The algorithm developed uses the step-doubling principle [10] and a predictor-corrector scheme [9]. The Jacobian is computed analytically and an appropriate strategy is required to stop the algorithm exactly at $\alpha = 1$.

There is no guarantee that an arbitrary initial estimate is connected to the solution by a smooth path that can be followed by the continuation algorithm. However, tests have indicated that convergence characteristics are greatly enhanced if all variables in the initial estimate are "synchronized". To obtain signal synchronization, ω_0 is set to a constant value in the region where oscillation is expected and a sinusoidal independent source of large amplitude is introduced in a suitable position in the circuit. For single FET oscillators, this source is applied to the gate of the FET. The resulting forced circuit is then solved by standard techniques, and the solution is used as an initial estimate for the autonomous problem. A different technique to obtain signal synchronization was discussed in [5].

Since convergence characteristics are generally improved when the number of variables is reduced, another useful technique is harmonic stepping. In this technique, the SB equation is initially solved for a small number of samples, and then this number is increased until the desired accuracy is attained. The solution of a problem is used as an initial estimate for the subsequent problem with the aid of sinusoidal interpolation. This technique was first suggested in [4]. Harmonic stepping is only employed when convergence problems occur. In this case, good results were obtained with only one intermediate step before solving the SB equation for the desired number of samples.

IV. EXAMPLE OF SIMULATION

The MESFET oscillator designed in [12] was simulated using the approach described in this paper. Fig. 2 shows the computed frequency of oscillation as a function of the number of samples considered. For more than 19 samples (9 harmonics), the frequency of oscillation did not vary significantly. In the remaining simulations, the number of samples was then held equal to 19. The relatively large errors in the computed frequencies for small numbers of samples are, to a certain extent, due to the calculation of derivatives using a second-order backward difference formula. Clearly, this formula does not yield an acceptable accuracy for a small number of samples. Calculating derivatives in the frequency domain will reduce this effect, although no appreciable difference is expected as the number of samples approaches the minimum required for good accuracy.

Gate and drain voltage waveforms for the original design are displayed in Fig. 3. The feedback circuitry used in the design comprises a series LC circuit between gate and drain. The oscillation frequency as a function of the value of the feedback inductor is in Fig. 4, and in all cases convergence was fast and smooth. In particular, as a consequence of signal synchronization, the paths followed by the pseudo arc-length continuation algorithm had simple shapes.

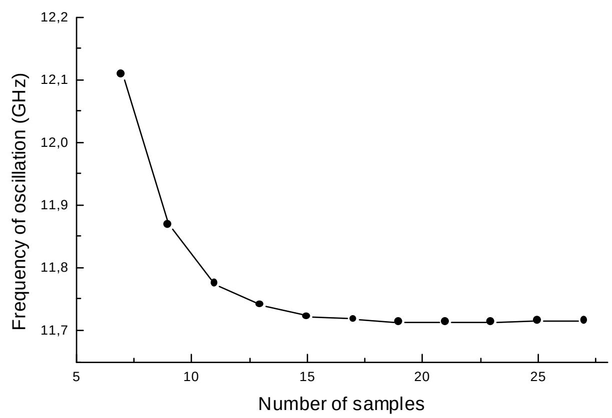

Fig. 2. Computed frequency of oscillation as a function of the number of time-domain samples considered.

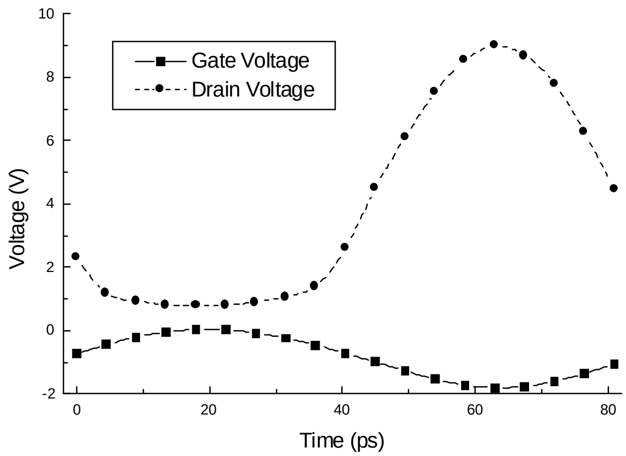

Fig. 3. Gate and drain voltage waveforms for an 1.5-nH feedback inductor.

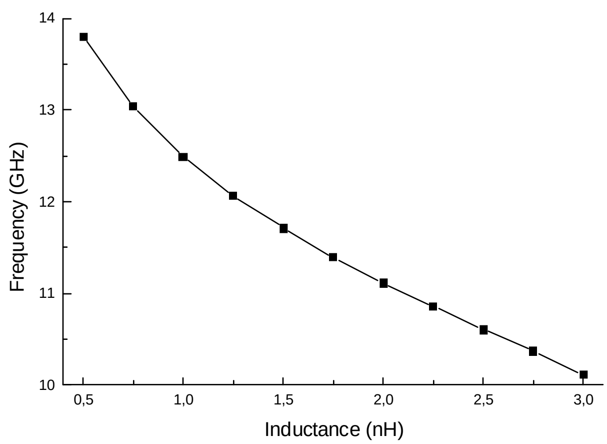

Fig. 4. Frequency of oscillation as a function of the value of the feedback inductor.

V. CONCLUSION

A new approach has been presented for the analysis of nonlinear autonomous circuits by frequency-domain methods. This approach provides effective answers to the problems of unwanted convergence to the DC solution and computation of the desired solution. The excellent convergence properties obtained were illustrated through the simulation of a MESFET oscillator.

Although implemented for Sample Balance, the approach described here can be readily adapted to Harmonic Balance analysis of nonlinear microwave autonomous circuits.

ACKNOWLEDGEMENTS

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